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1. Determine which of the following polynomials has $(x + 1)$ a factor:

(i) x^3+x^2+x+1

Solution:

$$\text{Let } p(x) = x^3+x^2+x+1$$

The zero of $x+1$ is -1 . [$x+1 = 0$ means $x = -1$]

$$p(-1) = (-1)^3+(-1)^2+(-1)+1$$

$$= -1+1-1+1$$

$$= 0$$

∴By factor theorem, $x+1$ is a factor of x^3+x^2+x+1

(ii) $x^4+x^3+x^2+x+1$

Solution:

$$\text{Let } p(x) = x^4+x^3+x^2+x+1$$

The zero of $x+1$ is -1 . [$x+1 = 0$ means $x = -1$]

$$p(-1) = (-1)^4+(-1)^3+(-1)^2+(-1)+1$$

$$= 1-1+1-1+1$$

$$= 1 \neq 0$$

∴By factor theorem, $x+1$ is not a factor of $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4+3x^3+3x^2+x+1$

Solution:

$$\text{Let } p(x) = x^4+3x^3+3x^2+x+1$$

The zero of $x+1$ is -1 .

$$p(-1) = (-1)^4+3(-1)^3+3(-1)^2+(-1)+1$$

$$= 1-3+3-1+1$$

$$= 1 \neq 0$$

∴ By factor theorem, $x+1$ is not a factor of $x^4+3x^3+3x^2+x+1$

(iv) $x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

Solution:

Let $p(x) = x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

The zero of $x+1$ is -1 .

$$\begin{aligned} p(-1) &= (-1)^3 - (-1)^2 - (2+\sqrt{2})(-1) + \sqrt{2} = -1 - 1 + 2 + \sqrt{2} + \sqrt{2} \\ &= 2\sqrt{2} \neq 0 \end{aligned}$$

∴ By factor theorem, $x+1$ is not a factor of $x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

2. Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$

Solution:

$p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$

$g(x) = 0$

$\Rightarrow x + 1 = 0$

$\Rightarrow x = -1$

∴ Zero of $g(x)$ is -1 .

Now,

$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= -2 + 1 + 2 - 1$

$= 0$

∴ By factor theorem, $g(x)$ is a factor of $p(x)$.

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$

Solution:

$p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$

$g(x) = 0$

$\Rightarrow x + 2 = 0$

$$\Rightarrow x = -2$$

\therefore Zero of $g(x)$ is -2 .

Now,

$$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$$

$$= -8 + 12 - 6 + 1$$

$$= -1 \neq 0$$

\therefore By factor theorem, $g(x)$ is not a factor of $p(x)$.

(iii) $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$

Solution:

$$p(x) = x^3 - 4x^2 + x + 6, g(x) = x - 3$$

$$g(x) = 0$$

$$\Rightarrow x - 3 = 0$$

$$\Rightarrow x = 3$$

\therefore Zero of $g(x)$ is 3 .

Now,

$$p(3) = (3)^3 - 4(3)^2 + (3) + 6$$

$$= 27 - 36 + 3 + 6$$

$$= 0$$

\therefore By factor theorem, $g(x)$ is a factor of $p(x)$.